

D. CVETKOVIĆ, P. ROWLINSON and S. SIMIĆ, *Eigenspaces of graphs*. Cambridge: Cambridge University Press, 1997, 258 p., prijs £45,- (hc) (Encyclopedia of mathematics and its applications; 66). ISBN 0-521-57352-1.

This book may be seen as the third in a series on spectral graph theory. The first, and excellent book *Spectra of graphs* by Cvetković, Doob and Sachs describes many of the relations between the combinatorial properties of graphs and the eigenvalues of their (0,1)-matrices. As is well-known, the spectrum of a graph in general does not determine the graph. This is one of the motivations to further study the structure of the eigenspaces of a graph, and this has, among others, resulted in the results collected in the book under review. Most of these results have appeared elsewhere, but not in book form.

Chapter 1 of the book contains many well-known results on graph spectra, and will be a "deja vu" for many readers. In Chapter 2, the authors describe how one can use eigenvectors to obtain results on (for example) the number of walks in a graph, switching, and the automorphism group of a graph. Also an elementary proof of Lloyd's theorem on perfect codes is outlined. Chapter 3 discusses some eigenvector techniques, including the Rayleigh quotient, to prove results on the eigenvalues of a graph, such as the maximum index (largest eigenvalue) in a class of graphs, and Fiedler's algebraic connectivity bound.

Motivated by the search for spectral invariants of graphs, the notion of graph angles (between eigenspaces and basis vectors) is introduced in Chapter 4, and used in Chapter 5 to study Ulam's reconstruction conjecture, and the construction of cospectral graphs. In Chapter 6, perturbations of graphs, such as adding or deleting edges or vertices, and their algebraic consequences are studied. Both the analytical and algebraic theory of matrix perturbations are used for this.

Chapters 7 and 8 contain results on so-called star partitions. These are partitions of the vertices of the graph, such that each part "naturally corresponds" to a basis of one of the eigenspaces of the graph. This concept has interesting combinatorial aspects. It is, for example, applied to domination in graphs. In Chapter 8 star partitions of graphs are used to study the graph isomorphism problem. For example, the authors give an alternative proof of the fact that isomorphism testing for graphs with bounded eigenvalue multiplicities can be done in polynomial time.

The final chapter contains miscellaneous results, including an application of graph angles to Hückel's theory in chemistry, and the impossibility of decomposing the complete graph on ten vertices into three Petersen graphs. The book closes in the same spirit as *Spectra of graphs*: it contains a 4-page table of graph angles of all connected graphs with at most five vertices.

Overall, I think this book is worth studying for people interested in using linear algebra in graph theory, and parts of it for those interested in the graph reconstruction and isomorphism problems.

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